Exercise 29

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval.

$$f(x) = x^3 - 100x + 2, \text{ between } x = 0.01 \text{ and } x = 0.1$$

[TYPO: Add a period after 0.1 to be consistent with the previous exercises.]

Solution

Plug $x = 0.01$ and $x = 0.1$ into the function.

$$f(0.01) = (0.01)^3 - 100(0.01) + 2 = 1.000001$$

$$f(0.1) = (0.1)^3 - 100(0.1) + 2 = -7.999$$

Since $f(x)$ is a polynomial function (a smooth and continuous function), $f(x)$ has to take on every value between -7.999 and 1.000001 for $0.01 < x < 0.1$ by the Intermediate Value Theorem. Therefore, $f(x)$ has a zero between $x = 0.01$ and $x = 0.1$.